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by

L.B. Freund
Division of Engineering
Brown University
Providence, RI 02912

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L. B. Freund
Division of Engineering
Brown University
Providence, RI 02912

ABSTRACT

Some concepts available for interpreting dynamic fracture phenomena are reviewed. These include the mechanical characterization of crack edge fields, energy variations associated with crack growth, and experimental observations relevant to the points raised. More recently developed and still incomplete ideas on the influence of crack tip plasticity, material strain rate sensitivity and three dimensional effects are also outlined.

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1. INTRODUCTION

Dynamic fracture is a branch of the engineering science of fracture mechanics concerned with fracture phenomena on a time scale for which inertial resistance of the material to motion is significant. The deformable body typically contains a dominant crack or other stress concentrating defect, and the phenomena of primary interest are those associated with conditions for the onset of extension of a crack or its arrest. Material inertia can have a significant effect in a variety of ways. Load transfer from the rapidly loaded boundary of a body to the region of a crack edge can occur by means of stress waves. Likewise, a rapidly running crack emits stress waves which can be geometrically reflected or scattered back to the region of the crack. It is through such waves that a rapidly running crack senses the nature of the imposed loading on the body through which it runs, as well as the configuration of the body. Material inertia may also lead to effects more subtle than those associated with load transfer. Crack tip fields are usually distorted from their equilibrium forms during rapid crack growth. Inertial resistance to motion on a very small scale near the edge of a crack may make the material appear more resistant to separation than it is due to its strength alone. There is a wide range of physical mechanisms by which materials separate on the scale of material microstructure and, in cases where multiple mechanisms are competing, inertial effects can have an influence on which is operative.

There are many facets to the study of dynamic fracture viewed as an area of basic research. Its theoretical underpinnings may be found among the basic concepts of continuum mechanics and materials science, and it has borrowed heavily from the theories of fracture under equilibrium conditions. A key element in the area is the identification of system parameters that characterize the resistance of materials to fracture and the measurement of these parameters for real materials. Experimental work in the area is extremely challenging because, typically, many observations must be made in a short period of time in a way which does interfere with the process itself. Crack tip data are difficult to extract from load point data due to the intervening stress waves and, furthermore, quantities of fundamental interest are not measurable directly but instead must be inferred indirectly through measurement of other quantities. A related point concerns the importance of developing a clear understanding of the connection between the values of fracture characterizing parameters and the physical mechanisms operative on the scale of microstructure in the material. It is only through such understanding that advances in the development of fracture resistant materials can occur. Finally, analytical and/or computational modelling of dynamic fracture phenomena has played a key role in developing insight into various phenomena, in providing a means for interpretation of data, and in studying the influence of competing effects in complex situations.

In the sections that follow, an overview of research in the mechanics of dynamic fracture is given, with emphasis on some emerging issues in this branch of fracture mechanics. Due to space constraints, important aspects of the field are not covered, particularly those concerned with the influence of material microstructure on macroscopic fracture response and with numerical simulation studies of dynamic fracture phenomena. Furthermore, experimental methods are given only a cursory treatment. Clearly, sustained progress in the field will require proper balance among all aspects of phenomena related to dynamic fracture of materials.

The fruits of research in this area have found application in studies of rapid crack propagation and crack arrest in pressure vessels and piping systems, cleavage crack growth in crystalline materials, dynamic earth faulting viewed as a fracture process, stress wave emission from growing cracks as a diagnostic tool in material evaluation, quantitative nondestructive inspection and evaluation of materials, and the erosion of material surfaces by high speed particulate or droplet impact. The area continues to be rich in challenging and potentially important problems. Some earlier reviews of the topic are presented in [1,2,3,4].

2. ELASTODYNAMIC CRACK TIP FIELDS

The first part of this article is concerned with infinitesimal deformation in a homogeneous and isotropic elastic material. All fields representing physical quantities are referred to a set of cartesian coordinates (x_1, x_2, x_3) or (x, y, z) fixed in the undeformed body. Standard index or vector notation is used. The displacement vector $\vec{u}$ satisfies

$$c_d^2 \nabla\nabla \cdot \vec{u} - c_s^2 \nabla \times \nabla \times \vec{u} = \partial^2 \vec{u}/\partial t^2 \tag{2.1}$$

where c_d and c_s are the propagation speeds of plane dilatational and shear waves, respectively, in the solid. This equation embodies Hooke's law and momentum balance, and it should be replaced by a suitable integral form when dealing with fields for which the derivatives do not exist. The Rayleigh wave speed is c_r.

General Concepts

The Helmholtz representation of the displacement vector in terms of the scalar dilatational potential ϕ and the vector shear potential $\vec{\psi}$ is introduced, namely,

$$\vec{u} = \nabla\phi + \nabla \times \vec{\psi}. \tag{2.2}$$

A displacement vector derived from potential functions according to (2.2) will satisfy (2.1) if the potentials satisfy the wave equations

$$c_d^2 \nabla^2 \phi - \partial^2 \phi/\partial t^2 = 0 \qquad c_s^2 \nabla^2 \vec{\psi} - \partial^2 \vec{\psi}/\partial t^2 = 0 \tag{2.3}$$

and if $\vec{\psi}$ is divergence free, $\nabla \cdot \vec{\psi} = 0$. The completeness of this representation has been proved by Sternberg [5]. In the case of plane strain or plane stress, the vector $\vec{\psi}$ has a single nonzero component in the direction perpendicular to the plane of deformation. Attention will be focussed here on the case of plane strain.

Consider plane strain deformation in the x_1, x_2–plane. Suppose that a planar crack occupies that part of the plane $x_2 = 0$ for which $x_1 < \ell(t)$. The nature of mechanical fields very close to the crack tip compared to distance to the nearest boundary or to the other end of the crack is of primary interest for the moment, so that it is adequate to view the crack as being semi-infinite and the body to be otherwise unbounded. The symmetries $u_1(x_1, x_2, t) = u_1(x_1, -x_2, t)$ and $u_2(x_1, x_2, t) = -u_2(x_1, -x_2, t)$ characterize the tensile opening mode of deformation, or mode I in the conventional terminology of fracture mechanics, for the cracked solid. Other independent modes are analyzed in much the same way as mode I. For points near the crack edge, it can be shown that the components of stress σ_{ij} have universal spatial dependence. In terms of polar coordinates r, θ centered on the moving crack tip with $\theta = 0$ coinciding with the x_1-axis,

$$\sigma_{ij} = \frac{K_I(t)}{\sqrt{2\pi r}} \Sigma_{ij}(\theta, v) + O(1) \quad \text{as} \quad r \to 0 \tag{2.4}$$

where $v = \dot{\ell}(t)$ is the instantaneous speed of the crack tip. The result (2.4) may be obtained by means of an interior asymptotic expansion of all fields about $r = 0$, and the angular variation $\Sigma_{ij}(\theta, v)$ is given explicitly by Freund and Clifton [6] for arbitrary $\dot{\ell}$ less than the Rayleigh wave speed. The angular variation of the circumferential tensile stress and the maximum shear stress for several crack tip speeds are shown in Fig. 2.1 and 2.2. The crack tip particle velocity likewise may be expanded in powers of r around the crack tip with the result that

$$\frac{\partial u_i}{\partial t} = -\frac{v K_I(t)}{E\sqrt{2\pi r}} U_i(\theta, v) + o(1) \quad \text{as} \quad r \to 0 \tag{2.5}$$

where E is the elastic modulus of the material.

Several comments should be made about the result (2.4). For one thing, the asymptotic analysis leading to it yields terms more singular than inverse square root, but (2.4) is the most singular contribution representing a state of bounded total energy, so the more singular terms are ruled out on physical grounds. Further, it is noted that the only feature of the near tip field to vary from one particular situation to another is the scalar multiplier $K_I(t)$, the so-called ***elastic stress intensity factor***, which contains information about the geometry of the body and the nature of the loading. The normalization of Σ_{ij} consistent with the common definition of stress intensity factor is $\Sigma_{22}(0, v) = 1$. The result thus generalizes the stress intensity factor concept of equilibrium fracture mechanics which was the cornerstone of Irwin's pioneering contributions in this area [7]. Finally, it must be recognized that a stress distribution which is singular at the crack tip is an abstract idealization. The rationale for admitting the singular stress distribution in the study of fracture of real materials is based on the *universal* spatial dependence of the crack tip stress field and on the concept of *small scale yielding*. The small scale yielding hypothesis presumes that the potentially large stresses in the vicinity of the crack edge are relieved through plastic flow, or some other inelastic process, throughout a region which has lateral dimensions that are small compared to the crack length and other body dimensions. Under these conditions, the stress distribution in the elastic material surrounding the inelastic crack tip zone is adequately described by the dominant singular term in the elasticity solution based on a sharp elastic crack. This surrounding field is completely determined by the stress intensity factor and, consequently, the stress intensity factor provides a one parameter characterization of the load level applied to the material in the inelastic crack tip zone. The stress intensity factor itself does not provide information on the way in which the material in this zone responds to the applied loading. While the actual size of any inelastic region can be determined only through detailed analysis of the full deformation fields, it is clear that the size must scale with the length $(K_I/\sigma_y)^2$ where σ_y is the tensile yield strength of the material.

Dynamic Stress Intensity Factor Solutions

In view of the central role of the stress intensity factor $K_I(t)$ as a characterizing parameter for mechanical fields near the edge of a crack, the relationship between K_I

and the applied loading and/or configuration of a solid containing a crack is important to understand. Analysis aimed at establishing this relationship for particular situations has become a significant part of fracture mechanics. Indeed, application of the general concepts in any particular case hinges on the ability to actually solve the mathematical problem that has been formulated to describe the process of interest. For stress wave loading of a stationary crack under two-dimensional conditions, a number of mathematical techniques have been developed for extracting the time-dependent stress intensity factor history. Among these methods are the approach based on integral transforms and the Wiener-Hopf technique pioneered by deHoop [8], the method of homogeneous solutions [9], the use of path-independent integrals of Laplace transformed fields [10], the dynamic weight function method [11], and superposition of moving dislocations [12]. Each of these methods has been applied in the analysis of the problem class being considered, namely, dynamic loading of a mode I crack under plane strain conditions. Various extensions have also been introduced. For example, Thau and Lu [13] have analyzed the diffraction of a tensile pulse by a *finite length* crack in order to assess the effect of the first diffracted wave on the local stress intensity factor.

In general, mathematical problems involving a characteristic length have been particularly unwieldy in this area and direct methods of analysis based on integral transforms or homogeneous solutions cannot be applied. Some solutions have been obtained by indirect methods, however. For example, the analysis by Freund [12] based on a dislocation superposition scheme yielded the exact transient stress intensity factor history for the case of an opposed pair of concentrated loads suddenly applied to the opposite crack faces at a fixed distance from the crack tip. A particularly interesting feature of the solution is that, after the Rayleigh wave generated at the load points reaches the crack tip, the stress intensity factor immediately takes on a constant value equal to its final equilibrium value.

The study of mechanical fields near the edge of an advancing crack in a nominally brittle solid was opened with the pioneering analysis of Yoffe [14]. She analyzed the case of a mode I crack of fixed length gliding steadily through a body subjected to uniform remote tension. While the problem is admittedly unrealistic, she drew conclusions based only on those features that did not depend on the fictitious crack length. The steady motion of a semi-infinite mode I crack was studied by Craggs [15] who noted that the asymptotic crack tip field was the same as that found by Yoffe. The problem of steady growth of a mode I crack along the centerline of an infinitely long strip subjected to uniform edge conditions was analyzed by Nilsson [16]. Some of the objections to steady-state crack growth analysis were overcome by Broberg [17], who analyzed the transient growth of a mode I crack from zero initial length at constant rate in a tensile field, and by Baker [18], who analyzed the transient extension of a semi-infinite mode I crack under the action of suddenly applied uniform crack face pressure. Some of the pioneering steps toward lifting the restriction to *constant* crack tip speed were taken by Kostrov [19] and Eshelby [20] in their work on the nonuniform extensions of cracks in the antiplane shear mode, or mode III in standard fracture mechanics terminology. They deduced exact stress intensity factor solutions for a variety of loading situations. A particularly interesting observation was made by Eshelby for nonsteady crack extension under time independent loading. He showed that if a crack propagates under these conditions and then suddenly stops, an *equilibrium field* is radiated out from the crack edge behind a wavefront traveling with the shear wave speed. Furthermore, this equilibrium field is the equilibrium field for the applied loads and instantaneous crack length, a truly remarkable result for a two-dimensional wave propagation field.

Kostrov based his analysis on the Volterra integral representation of the solution of the elementary wave equation is two dimensions, whereas Eshelby applied an obscure theorem concerned with the electromagnetic radiation from a nonuniformly moving line charge. Neither technique could be carried over to the case of mode I, but Eshelby's solution provided a clue on a way to proceed. If it could be shown that an equilibrium field radiated out when a growing mode I crack suddenly stops, then a complete solution for nonuniform motion could be built up as a sequence of many short start/stop segments. Because of the presence of free surface Rayleigh waves, it was most unlikely that the same strong result would carry over to the plane strain case. On the other hand, it was noted that a weaker result would suffice, namely, that an equilibrium field radiated out along the prospective fracture path when the crack stopped. It was established by Freund [21] that this is indeed the case for time independent loading, and the result led to the exact stress intensity for nonuniform crack growth under time-independent loading. Similar results were subsequently obtained for time-dependent loading [22,23]. The general result is summarized as follows: The stress intensity factor for mode I extension of a half plane crack is given by a universal function of instantaneous crack tip speed $k(\dot{\ell})$ times the stress intensity factor appropriate for a crack of fixed length, equal to the instantaneous length, subjected to the given applied loading, whether this loading is time independent or time dependent. That is, the stress intensity factor K_I is given by

$$K_I(t, \ell, \dot{\ell}) = k(\dot{\ell})\, K_I(t, \ell, 0) \tag{2.6}$$

The function describing k has a complicated form, but simple behavior. It decreases monotonically from $k(0) = 1$ to $k(c_r) = 0$. A function providing a reasonable approximation is $k(v) = (1 - v/c_r)/(1 - 0.5v/c_r)$. The result (2.6) was verified and extended by means of more direct procedures by Kostrov [24] and Burridge [25].

Critical Stress Intensity Factor Criterion

The main implication of the observed role of the stress intensity factor as a characterizing parameter is the following. Consider two bodies of the same material, but having different shapes and/or having cracks of different size. Suppose that the two bodies are loaded to result in the same mode of crack tip deformation (mode I, in the present case). If the loading results in the same stress intensity factor in the two cases, then the material in the crack tip region is assumed to respond in the same way in the two cases. The foregoing idea is exploited in engineering practice, for example, by measuring the stress intensity factor at which a crack will begin to advance in a well-characterized laboratory specimen, and then assuming that a cracked structure will experience crack growth at the same level of stress intensity.

The engineering science of linear elastic fracture mechanics (LEFM), which has evolved from this idea, has been profitably extended to situations in which material inertia plays a significant role. Given the significance of the stress intensity factor as a characterizing parameter for each mode of crack opening, a simple criterion for the onset of crack growth is the following: A crack will begin to extend when the stress intensity factor has been increased to a material specific value, usually called the *fracture toughness* of the material and commonly denoted by K_{Ic} for mode I plane strain deformation. For values of the stress intensity factor smaller than the critical value there is no growth, and values larger than the critical value are inaccessible. This is the Irwin criterion of LEFM in it simplest form. It should be noted that such a criterion is a physical postulate on material response, on the same level as the stress-strain relation or other physical postulate on which the mathematical formulation is based. In the statement of this criterion, it should be emphasized that K_{Ic} is a material parameter and that $K_I(t)$ is a feature of the stress field. The foregoing statement of the Irwin criterion may be applied without modification to the study of fracture initiation in nominally elastic bodies subjected to stress wave loading and dynamic crack propagation. In its simplest form, it has been assumed that a crack edge will be stationary if $K_I(t) < K_{Ic}$ for any loading history, but that the crack will grow with some speed $\dot{\ell}(t) > 0$ if $K_I(t) = K_{Ic}$. Values of stress intensity factor greater than K_{Ic} are inaccessible. The criterion has been further generalized by hypothesizing that that the critical level of stress intensity required to drive the fracture depends on the instantaneous crack tip speed $\dot{\ell}(t)$ or possibly other instantaneous values of system parameters or their histories. The dependence of the critical stress intensity factor on speed is sometimes denoted by $K_{Id}(\dot{\ell})$, although the notation is not standard, and the material response represented by K_{Id} versus its arguments is called the *dynamic fracture toughness.*

3. TRANSIENT STRESS INTENSITY FACTOR HISTORY

In this section, the matter of actually calculating the transient stress intensity factor history as a property of the stress distribution is discussed. Consider a body of elastic material that contains a half plane crack but that is otherwise unbounded. In the present instance, the material is stress free and at rest everywhere for $t < 0$, and the crack faces are subjected to uniform normal pressure of magnitude σ^* for $t \geq 0$. For points near to the crack face compared to distance to the crack edge, the transient field consists only of a plane wave parallel to the crack face and traveling away from it at speed c_d. As this plane dilatational wavefront passes a material point in its path, the component of stress σ_{yy} (or σ_{22}) changes discontinuously from zero to $-\sigma^*$ and the particle velocity $\partial u_y/\partial t$ (or $\partial u_2/\partial t$) changes discontinuously from zero to $\pm\sigma^*/\rho c_d$ for $\pm y > 0$.

Near the crack edge, on the other hand, the deformation field is more complex. A nonuniform scattered field radiates out from the crack edge behind a cylindrical wavefront (circular in two dimensions) of radius $c_d t$ that is centered on the crack edge. Due to the coupling of dilatational and shear waves at a boundary, this scattered field also includes a cylindrical shear wavefront of radius $c_s t$ that is centered on the crack edge, plus the associated plane fronted headwaves traveling at speed c_s.

The process being described here involves neither a characteristic length nor a characteristic time. Thus, the components of stress and particle velocity are homogeneous functions x, y, t of degree zero. An immediate consequence of (2.4) is that, for points very close to the crack edge compared to the distance to the nearest wavefront, say,

$$\sigma_{yy}(x,0,t) \approx C_I \sigma^* \sqrt{\frac{c_d t}{x}}, \qquad \frac{c_s t}{x} \ll 1 \tag{3.1}$$

where C_I is an undetermined dimensionless constant. In view of (3.1), the tensile stress intensity factor is also known up to the constant C_I, that is,

$$K_I(t) = \lim_{x \to 0^+} \sqrt{2\pi x}\,\sigma_{yy}(x,0,t) = C_I \sigma^* \sqrt{2\pi c_d t} \tag{3.2}$$

Through a solution of the transient boundary value problem following the method of deHoop [8], it is found that the dimensionless constant has the value $C_I = \sqrt{2(1-2\nu)}/\pi(1-\nu)$ where ν is Poisson's ratio.

The stress intensity factor (3.2) was obtained for the case of crack face loading. However, it may be given another interpretation for the linear system. Consider again the same configuration, but with the crack faces free of traction. Suppose that a plane tensile pulse propagates toward the crack plane at speed c_d. The front of the pulse is parallel to the crack plane, and it carries a jump in stress σ_{yy} from its initial value of zero to $+\sigma^*$. The pulse arrives at the crack plane at time $t = 0$, and it is partially reflected and partially scattered, or diffracted, upon reaching the crack. Aside from the uniform plane wave, the field of the diffraction process is identical to that for the suddenly applied crack face pressure. In particular, the relationship between the stress intensity factor and the loading magnitude σ^* is identical in the two cases.

Suppose that the crack edge remains stationary at $x = \ell(t) = 0$ for some time after the load begins to act, but at some later time $t = \tau > 0$ the crack edge begins to advance in the x–direction at nonuniform speed $\dot{\ell}(t) < c_r$. The time τ is called the ***delay time.*** An exact stress intensity factor solution for this situation has been provided by Freund [22], with the result that

$$K_I(t) = \lim_{x \to 0^+} \sqrt{2\pi x}\,\sigma_{yy}(x,0,t) = k(\dot{\ell}) C_I \sigma^* \sqrt{2\pi c_d t}\,. \tag{3.3}$$

This result differs from the corresponding result for a stationary crack only through the dimensionless factor $k(\dot{\ell})$ which is the universal function of instantaneous crack tip speed introduced in (2.6).

Now, consider a tensile rectangular stress pulse of magnitude σ^* and duration t^* normally incident on a traction free crack. If the crack does *not* extend, then it is clear from (3.3) that the crack tip stress intensity factor will increase in proportion to $\sqrt{t}$ for $0 < t < t^*$, and it will decrease in proportion to $\sqrt{t} - \sqrt{t - t^*}$ for $t^* < t < \infty$. The variation of $K_I(t)$ without extension is shown in Fig. 3.1 as the solid line. The largest value of stress intensity factor without extension is $K_I(t^*) = C_I \sigma^* \sqrt{2\pi c_d t^*}$, so the crack will grow only if $K_I(t^*) > K_{Ic}$. It is assumed that this is the case, and the simplest possible fracture criterion, namely,

$$\dot{\ell}(t) = 0 \text{ with } K_I(t) < K_{Ic} \quad \text{or} \quad \dot{\ell}(t) \geq 0 \text{ with } K_I(t) = K_{Ic} \tag{3.4}$$

is adopted. It is observed that the fracture criterion is first satisfied at time $t = t_i$ where

$$t_i = \frac{1}{2\pi c_d} \left(\frac{K_{Ic}}{C_I \sigma^*} \right)^2 . \tag{3.5}$$

This time is called the *incubation time*, and the crack will grow for $t > t_i$. The fracture criterion imposes the condition that

$$K_{Ic} = k(\dot{\ell}) C_I \sigma^* \sqrt{2\pi c_d} \begin{cases} \sqrt{t} & \text{if } t_i < t \leq t^* \\ \sqrt{t} - \sqrt{t - t^*} & \text{if } t^* < t \leq t_a \end{cases} \tag{3.6}$$

where the function k was introduced in (2.6) and

$$t_a = (t_i + t^*)^2 / 4t_i \tag{3.7}$$

is the arrest time, that is, the time at which the decreasing $K_I(t)$ passes the value K_{Ic}; see Fig. 3.1. The relationship (3.6) is an ordinary differential equation for the position of the crack tip $\ell(t)$ as a function of time and, as such, it is an *equation of motion* for the crack tip, analogous to the equation of motion for a particle in elementary mechanics. This equation is subject to the initial condition $\ell(0) = 0$ and it applies during $t_i < t < t_a$.

The features of the solution of (3.6) are evident. The crack tip begins to move at time $t = t_i$, it accelerates for $t_i < t < t^*$, it decelerates for $t^* < t < t_a$, and it arrests at time t_a. This response is depicted in Fig. 3.2. Some features of (3.6) that are general and common to virtually all elastodynamic crack models are noted. For one thing, the equation of motion is a first-order ordinary differential for crack position as a function of time. If the analogy with particle mechanics is pursued, this implies that the coefficient of the acceleration term in the equation is zero, or that the crack has *no effective mass.* This should not be interpreted as evidence that inertial effects are not important in the phenomenon. It simply means that the crack velocity, rather than acceleration, varies directly with the driving force. It also implies that the crack velocity may change discontinuously in time without violating any physical laws. The equation of motion also indicates that, if the duration of the loading pulse is indefinitely long, then the crack tip speed increases continuously toward c_r, the Rayleigh wave speed of the material. In this sense, the Rayleigh wave speed is the theoretical terminal velocity of a crack tip. In most actual crack growth situations, other effects intervene before the crack speed approaches the theoretical limiting velocity. Finally, if a material interface with no strength is characterized by the condition that $K_{Ic} = 0$, then the equation of motion indicates that a dynamic separation point propagates along the interface only at the speed c_r.

Three Dimensional Stress Intensity Factor History

A procedure for determining the stress intensity factor histories for a class of three dimensional elastodynamic crack problems was recently introduced [26]. The geometrical configuration is a half plane crack in an otherwise unbounded body, a configuration which is invariant under translation in the direction of the crack edge. However, the crack faces are subjected to tractions that result in a true three dimensional stress wave field and in a variation of the transient stress intensity factor along the edge of the crack. The method is based on integral transform methods with a special interpretation of the transformed fields.

The procedure leads to exact results for stress intensity factor histories; unfortunately, it is not possible to extract full three dimensional solutions. The problem used to introduce the technique was the sudden application of a symmetric pair of line loads to the crack faces, with the direction of the line being *perpendicular* to the edge of the crack. The opposed line loads tend to open the crack, producing a mode I stress intensity factor history at each point along the edge. The method has been extended to attack the problem of moving loads on the crack faces by Ramirez [27] and the problem of transient crack growth at constant speed through a time independent stress field by Champion [28].

4. ENERGY VARIATIONS DURING DYNAMIC CRACK GROWTH

Interest in the mechanical energy that is extracted from a deformable solid during crack advance is an outgrowth of Griffith's original hypothesis that energy has to be supplied to create new surface. A crack tip contour integral expression for dynamic energy release rate was proposed by Atkinson and Eshelby [29], who argued that the form for dynamic growth should be the same as for quasi-static crack advance with the elastic energy density replaced by the total internal energy density. The equivalent integral expression for dynamic energy release rate in terms of crack tip stress and deformation fields was subsequently derived directly from the field equations of elastodynamics by Kostrov and Nikitin [30] and by Freund [31]. The result was obtained by enforcing instantaneous energy rate balance for the time-dependent region inside the boundaries of the body but outside of a small loop surrounding and translating with the crack tip. By application of Reynolds' transport theorem and the divergence theorem, an expression for the crack tip energy flux through the small crack tip loop in the form of a line integral along the loop. A more general result, including this elastodynamic energy release rate expression as a special case, may be derived for arbitrary material response.

Consider crack growth in a two dimensional body in the x_1, x_2-plane, with the crack in the plane $x_2 = 0$ and crack growth in the x_1−direction at instantaneous speed v. A small contour Γ begins on one face of the crack, surrounds the crack tip, and ends on the opposite face of the crack. The contour is fixed in size and orientation, and it translates with the crack tip. As it does so, it sweeps out a tubular surface in space-time as the crack grows. The steps outlined by Nakamura, Shih and Freund [32] lead to the result that

$$F(\Gamma) = \int_{\Gamma} \left(\sigma_{ij} n_j \frac{\partial u_i}{\partial t} + (U + T) v n_1 \right) d\Gamma \tag{4.1}$$

is the instantaneous rate of energy flow out of the body through Γ, where T is the kinetic energy density and U is the stress work density for ***any material response***, namely,

$$U = \int_0^t \sigma_{ij} \frac{\partial^2 u_i}{\partial t' \partial x_j} dt', \qquad T = \tfrac{1}{2} \rho \frac{\partial u_i}{\partial t} \frac{\partial u_i}{\partial t} \tag{4.2}$$

and ρ is the mass density of the material. The first term in (4.1) is the rate of work of the material outside of Γ on the material inside Γ, and the second term represents the energy flux through Γ due to mass transport associated with crack motion. The general result (4.1) underlies virtually all crack tip energy integrals that have been defined and applied in fracture mechanics, in the sense that each is obtained from (4.1) by invoking appropriate restrictions on material response (through U) and on crack tip motion. For example, for equilibrium fields in nonlinear elastic material, it leads to Rice's J−integral [33]. For steady crack growth, it leads to a path-independent integral considered in various aspects by Hutchinson [34] and Willis [35]. The implications of (4.1) for elastodynamic crack growth will be considered next, and application in other situations will be considered subsequently.

For a linear elastic material, $U = \frac{1}{2}\sigma_{ij} u_{i,j}$ and the energy released from the body per unit crack advance in the x_1−direction is the limit of $F(\Gamma)/v$ as Γ is shrunk onto the crack tip. It is obvious from the hyperbolic character of the governing equations that $F(\Gamma)$ cannot be a path-independent integral, in general. The limit is usually termed the *dynamic energy release rate* and is denoted by G. For the concept to have any fundamental significance, it is necessary that the limit be independent of the shape of Γ, and this was demonstrated for the near tip elastodynamic field by Freund [31]. Because the near tip distribution of mechanical fields is known explicitly for elastic response, the relationship between G and the stress intensity factor may be established by evaluating the integral (4.1), with the result that

$$G = \frac{1-\nu^2}{E} A(v)\, K_I^2 \tag{4.3}$$

where ν and E are the elastic constants of an isotropic solid and $A(\cdot)$ is a universal function of the *instantaneous* crack tip speed v. The function A has the properties that $A(0) = 1$, $A'(0) = 0$ and $A(v) \to \infty$ as $v \to c_r^-$. Thus, $G > 0$ for $0 < v < c_r$. The result (4.2) applies for all mode I elastodynamic crack growth situations. Under the special condition that the complete elastic field is time-independent as seem by an observer moving with the crack tip, the integral $F(\Gamma)$ is path-independent [29,36].

A specimen configuration that has been employed for crack propagation experiments is the split rectangular plate known as the double cantilever beam. Because of its configuration, the specimen has been analyzed by a strength of materials approach, whereby the arms of the specimen are viewed as beams cantilevered at the crack tip end (and thus the common name for it). The deformation and associated stress distribution obtained by applying the usual assumptions of beam theory represent internally constrained plane strain or plane stress fields, and the general energy flux expression may be applied directly to compute energy release rate. Suppose that the plane of deformation is the x, y−plane, that the crack is in the plane $y = 0$, and that the tip is at $x = \ell(t)$. Then the contour Γ in the plane of deformation begins on one traction free crack face at $x = \ell(t) - 0$. It runs along a cross section of one beam arm to the traction free boundary at $y = h$, along this boundary until $x = \ell(t) + 0$, then along the cross section to $y = -h$, and it returns to the other crack face in such a way that Γ has reflective symmetry with respect to the crack plane. Invoking the standard assumptions for an elastic Bernoulli Euler beam, it can be shown through a direct application of (4.1) that the energy release rate is

$$G = 12(1-\nu^2)\, M(\ell,t)^2/Eh^3 \tag{4.4}$$

where $M(\ell,t)$ is the bending moment per unit thickness in each beam arm at the crack tip end. If it is argued, as in the case of compliance methods of equilibrium fracture mechanics, the the energy being supplied to the crack tip cross section is actually being absorbed at the crack tip, then (4.2) yields an expression for the crack tip stress intensity factor for the specimen in terms of the internal bending moment at $x = \ell(t)$, namely,

$$K_I(t) = M(\ell,t)\sqrt{12/A(\dot{\ell})h^3}\,. \tag{4.5}$$

At the level of the beam approximation, the singularity at the crack tip on the fracture plane is a concentrated force. On the level of the plane elastic field, however, it is the standard square root singular stress distribution. A number of similar connections between strength of materials models and elastic field models were established in the article by Freund [4].

It is noted briefly that because the tip of a growing crack is a sink of mechanical energy, the standard proof of uniqueness of solutions for elastodynamics requires modification. The uniqueness theorem is normally proved by showing that the rate of change of mechanical energy for a difference solution is zero, so that if the difference solution satisfies homogeneous initial and boundary conditions and the strain energy is positive definite, then the difference solution remains zero. For a difference solution for a running crack problem, the rate of change of mechanical energy can be shown to be equal to $-G$ [6]. Therefore, the uniqueness theorem is easily extended to such problems if $0 < v < c_r$. However, solutions for crack speeds in the range $c_r < v < c_s$ are not necessarily unique because $G < 0$ there.

The energy "lost" from a solid in a fracture mechanics description of dynamic crack growth is not actually lost but, instead, it is simply not accounted for in the continuum description of the process. A range of mechanisms of energy absorption at a crack tip can be identified. For example, in simple cleavage of covalently bonded crystals, the energy required to separate adjacent planes of atoms is roughly $1\, J/m^2$. During cleavage of crystals with ionic bonding or body centered cubic metallic crystals, crack growth is usually accompanied by highly localized dislocation motion in the material and apparent energies of fracture are on the order of $10\, J/m^2$. For "cleavage" of polycrystalline iron or "brittle" fracture of PMMA or similar polymer, a large portion of the energy consumed in the process of fracture is dissipated in the necking down of ductile ligaments left behind as the brittle fracture front advances through the material. In this case, the energy of fracture may be on the order of $10^3\, J/m^2$. For a high strength steel or aluminum alloy, crack advance usually occurs by nucleation of microvoids in regions of high stress triaxiality in the material and their ductile growth to coalescence. The energy dissipated in this process may be as low as $10^4\, J/m^2$, and it may be much higher. Finally, for crack growth in very ductile materials and/or in thin sections with minimal triaxial constraint, crack advance occurs by through-the-thickness shearing and the fracture energy may be almost indefinitely large. The classification of mechanisms could be modified or refined in any number of ways, of course.

A One-Dimensional Crack Growth Model

Analytical models of dynamic crack growth involving a single spatial dimension have been developed in connection with dynamic fracture toughness testing by Kanninen [37], Bilek and Burns [38] and Freund [39], in connection with seismic source modeling by Knopoff et al [40] and Landoni and Knopoff [41], and in connection with peeling of a bonded layer by Hellan [42]. Some of these models have been remarkably successful in enhancing insight into the particular physical process of interest, and the few models for which complete mathematical solutions exist provide the rare opportunity to see all aspects of a dynamic crack growth event in a common and relatively transparent framework. The discussion here will focus on the model developed by Freund [39] in order to illustrate the influence of reflected waves on crack growth in a double cantilever beam fracture specimen by means of a simple shear beam model of the specimen. The model may be rephrased in terms of the dynamics of an elastic string, an even simpler conceptual model, and it has been analyzed with numerous variations from this point of view by Burridge and Keller [43].

Consider a stretched elastic string lying along the positive x-axis. The string has mass per unit length ρ and characteristic wave speed c. The transverse deflection is $w(x,t)$, the elastic strain from the undeflected configuration is $\gamma = \partial w/\partial x$ and the transverse particle velocity is $\eta = \partial w/\partial t$. Initially, the string is free of transverse loading in the interval $0 < x < \ell_o$ and is bonded to a rigid, flat surface for $x > \ell_o$. A boundary condition in the form of a condition on $w(0,t)$, $\gamma(0,t)$, or a linear combination of the two is required and specific cases will be considered. Finally, suppose that the string is initially deflected but at rest, so that $w(x,0) = w_o(x)$ and $\eta(x,0) = 0$ where $w_o(x)$ is specified. At time $t = 0$, the string begins to peel away from the surface, so that at some later time $t > 0$ the free length is $0 < x < \ell(t)$. Thus, the field equations are to be satisfied in the time-dependent interval $0 < x < \ell(t)$ subject to the stated initial and boundary conditions. If $\ell(t)$ is specified, then the solution of the governing differential is subject to the "crack tip" condition that $w\big(\ell(t),t\big) = 0$ or, in rate form, that $\dot{\ell}\gamma + \eta = 0$ at $x = \ell(t)$. Actually, the field equations and boundary conditions can be satisfied for a host of crack motions $\ell(t)$. If a crack growth criterion is specified at $x = \ell(t)$, then a part of the solution procedure is to find that particular crack motion $\ell(t)$ for which the growth criterion is satisfied pointwise in time. Some cases of crack motion will be discussed after the issues of crack tip singular field and energy release rate have been considered within the framework of the dynamic string model.

Ahead of the crack tip the strain and particle velocity are clearly zero, whereas behind the tip they have nonzero values, in general. Thus, the crack tip carries a propagating discontinuity in strain and particle velocity. Because the equation governing motion of the string is the elementary wave equation, discontinuities propagate freely only at the characteristic wave speed of the string c. But the speed of the crack tip $\dot{\ell}(t)$ is not restricted to be always equal to c, so that the propagating crack tip must carry a momentum source or sink. In view of the fact that the only degree of freedom is the transverse deflection, the momentum source must be a generalized force that is work conjugate to w, namely a concentrated force acting at $x = \ell(t)$ and travelling with that point. This is the crack tip singular field for this simple structure. The elastic energy density and kinetic energy density for the string are $\frac{1}{2}\rho c^2\gamma^2$ and $\frac{1}{2}\rho\eta^2$. Application of the energy flux integral (4.1) for a contour surrounding the portion of the string at $x = \ell(t)$ yields

$$F = G\dot{\ell} = -\rho c^2\gamma\eta - \tfrac{1}{2}\rho c^2\gamma^2\dot{\ell} - \tfrac{1}{2}\rho\eta^2\dot{\ell} \tag{4.6}$$

and, in light of the kinematic boundary condition at $x = \ell(t)$,

$$G = \rho c^2(1 - \dot{\ell}^2/c^2)\gamma(\ell, t)^2 \tag{4.7}$$

Other ways to derive the result (4.7) are outlined by Freund [44].

The crack growth criterion that is adopted here for purposes of illustration is the energy balance criterion, according to which the crack must propagate in such a way that the energy release rate G is always equal to the specific fracture energy G_c, a material characterizing parameter that is assumed to be constant for simplicity. Consider equilibrium fields for the moment. If the boundary condition at $x = 0$ is that the displacement is held fixed at the level w^* (the fixed grip condition), then the uniform strain for any value of ℓ is w^*/ℓ and the elastic energy stored in the structure is $\frac{1}{2}\rho c^2(w^*)^2/\ell$. There is no external potential energy in this case. The energy consumed in fracture, on the other hand, is $G_c(\ell - \ell_o)$. The total energy thus has a stationary point at $\ell^2 = \rho c^2 (w^*)^2 / 2G_c$ corresponding to an equilibrium state that is also a state of incipient fracture. If the initial length ℓ_o is greater than this critical length for a given w^* then this value of imposed displacement is not large enough to induce crack growth. If, on the other hand, the initial length ℓ_o is less than this critical length for a given w^* then the energy released in a small excursion of the system from its initial configuration under equilibrium conditions exceeds the energy consumed in the fracture process, so that a state of equilibrium cannot be maintained. Inertial effects will be called into play to balance overall momentum of the system.

As a second example, suppose that the boundary condition at $x = 0$ is that the transverse force is held fixed at the level $\rho c^2 \gamma^*$ (dead weight loading). The uniform strain for any value of ℓ is then γ^* and the elastic energy stored in the structure is $\frac{1}{2}\rho c^2 (\gamma^*)^2 \ell$. The external potential energy is $-\rho c^2 (\gamma^*)^2 \ell$. As before, the energy consumed in fracture is $G_c(\ell - \ell_o)$. The system is thus in equilibrium and at a point of incipient crack growth if $(\gamma^*)^2 = 2G_c/\rho c^2$. If the imposed value of γ^* is not large enough in magnitude to satisfy this critical condition, then crack growth will not occur. If, on the other hand, the magnitude is too large to satisfy the critical condition, then equilibrium conditions cannot be maintained under any circumstances and, again, inertial effects will be called into play. Other boundary conditions may be considered for this simple system, but these two cases illustrate the circumstances under which inertial effects become significant in a crack growth process. For example, the influence of loading chain stiffness may be pursued by specifying a linear relationship between load and displacement at $x = 0$ [44].

Consider the former case (fixed grip condition). However, suppose that the crack tip is initially slightly blunted so that the level of applied end displacement w^* is greater than that required to initiate growth of a sharp crack with initial length ℓ_o. To be specific, suppose that the initial energy release rate $G^* = \rho c^2 (w^*/\ell_o)^2$ exceeds the level required to sustain growth of a sharp crack by a factor $n > 1$. Given a value of this factor, called the bluntness parameter, the relation $n = G^*/G_c$ actually specifies the end displacement w^* which must be imposed to initiate dynamic crack growth. The crack propagates with $G = G_c$, or $\dot{\ell} = 0$ if $G < G_c$.

From a solution of the differential equation governing motion for arbitrary $\ell(t)$ by the method of characteristics, say, it can be shown that $\gamma(\ell, t) = \gamma_o/(1 + \dot{\ell}/c)$ up until the wave emitted from the crack tip at the onset of growth reflects back onto the tip from the fixed end of the string. The crack propagation condition then requires that

$$G_c = G^*(1 - \dot{\ell}^2/c^2)/(1 + \dot{\ell}/c)^2 . \tag{4.8}$$

This is again an equation of motion for the crack tip deduced from a physical postulate on the nature of the fracture process. If G^* is eliminated in favor of n, it is found that the crack speed has the constant value $(n-1)/(n+1)$ before the first reflected wave overtakes the crack tip. If the procedure is pursued one step further, it is found that the first reflected wave reduces the crack tip strain to a level below that required to sustain crack growth. The crack thus arrests instantaneously, and arrest is accompanied by a negative jump in the crack tip energy release rate. The arrest length of the crack is $n\ell_o$, which is substantially larger than the equilibrium length of $\sqrt{n}\ell_o$ for the specified displacement w^*. Other situations involving G_c dependent on crack speed were considered in [39].

Because of the simplicity of the solution, the strain energy and kinetic energy as functions of crack length can be calculated easily, and a typical result is shown in Fig. 4.1 for $n = 4$. For this case, the crack speed up to arrest is $3c/5$, the arrest length is $4\ell_o$, and the crack length when the unloading wave reflects from the end $x = 0$ is $8\ell_o/5$. The interpretation of the energy variations is straightforward. At the instant of fracture, the strain magnitude at the crack tip is reduced from the supercritical value to the value necessary to satisfy the fracture criterion, and this reduction in strain implies a reduction in strain energy E_U. Because of the rate of this reduction in strain, however, the inertia of the material comes into play. The value of strain magnitude is reduced behind the wave traveling from the tip to the fixed end of the string, so the strain energy decreases and the kinetic energy E_T increases as the wave engulfs more and more of the string length. Because energy is being drawn from the body at the crack tip, the decrease in E_U is not balanced by the increase in E_T. When the stress wave reflects from the fixed end $x = 0$, the fixed displacement condition requires that it do so in just the right way to cancel the particle velocity. Thus, as the wave reflects back onto itself, the particle velocity η is reduced to zero and the strain is further reduced behind the reflected wave. Thus, the kinetic energy decreases after wave reflection, and the strain energy decreases but at a slower rate than before reflection. It should be noted that until the reflected wave overtakes the crack tip, the crack response is as though it were propagating in an unbounded body, that is, the crack tip shows no influence of the boundary at $x = 0$. The first influence of the fixed boundary on the crack tip arrives with the reflected wave which, as already noted for this simple structure, causes an instantaneous arrest. In effect, by the time that the stress wave communicates to the crack tip that the applied end displacement w_*^* is appropriate to maintain a certain equilibrium length for a sharp crack, the crack has already grown to a length greater than this equilibrium length. The post arrest state is thus sub-critical.

The string model does not provide an adequate model for analysis of dynamic fracture in real structures. However, the physical insight developed through study of simple and transparent models is helpful in considering more complex situations or in devising approximate models.

5. PLASTICITY EFFECTS IN DYNAMIC CRACK PROPAGATION

In cases where the extent of plastic flow is sufficiently great to preclude the small scale yielding assumption, or where the phenomenon of interest is exhibited on the scale of the crack edge plastic zone, the post yield response of the material must be taken into account. Few general results have been obtained for dynamic crack growth in nonlinear materials. Some recent studies are described in this section.

An Exact Result for Antiplane Shear

In an effort to explain the dependence of dynamic fracture toughness on crack tip speed observed in experiments on a high strength steel, the steady-state growth of a crack at speed v in the the antiplane shear mode, or mode III in fracture mechanics terminology, under small scale yielding conditions was analyzed by Freund and Douglas [45] and by Dunayevsky and Achenbach [46]. The field equations governing this process include the equation of momentum balance, the strain-displacement relations, and the condition that the stress distribution far from the crack edge must be the same as the near tip stress distribution in a corresponding elastic problem. For elastic-ideally plastic response of the material, the stress state is assumed to lie on the Mises yield locus, a circle of radius τ_o in the plane of rectangular stress components, and the stress and strain are related through the incremental Prandtl-Reuss flow rule. The material is linearly elastic with shear modulus μ outside of plastically deforming regions.

With a view toward deriving a theoretical relationship between the crack tip speed and the imposed stress intensity factor required to sustain this speed according to a critical plastic strain crack growth criterion, attention was focussed on the strain distribution on the crack line within the active plastic zone, and the influence of material inertia on this stress distribution. It was found that the distribution of shear strain on this line, say $\gamma_{yz}(x, 0)$ in crack tip rectangular coordinates, could be determined *exactly* in terms of the plastic zone size r_o in the implicit form

$$\gamma_{yz}(x,0) = \frac{\mu}{\tau_o}\left\{1 - \left(\frac{1-m^2}{2m^2}\right) \ell n \left(\frac{1-m^2h^2}{1-m^2}\right)\right\}$$
$$x = r_o \frac{I(-h)}{I(m)}, \qquad I(t) = \int_0^{(1-t)/(1+t)} \frac{s^{(1-m)/2m}}{(1+s)} ds \tag{5.1}$$

where $m = v/c_s$.

The exact result (5.1) resolved a long standing paradox concerning mode III crack tip fields. Rice [47] showed that the near tip distribution of strain $\gamma_{yz}(x,0)$ for steady growth of a crack under equilibrium conditions was singular as $\ell n^2(x/r_o)$ as $x/r_o \to 0$. On the other hand, Slepyan [48] showed that the asymptotic distribution for any $m > 0$ was of the form $(m^{-1} - 1)\,\ell n\,(x/r_o)$ as $x/r_o \to 0$. These two features could be verified by examining the behavior of the exact solution for dynamic growth (5.1) under the condition $m \to 0$ for any nonzero value of x/r_o and under the condition that $x/r_o \to 0$ for any nonzero value of m, respectively. The resolution of the paradox was found, however, in the observation that Slepyan's asymptotic solution is valid only if

$$\left(x/r_o\right)^{2m/(1+m)} \ll 1 \tag{5.2}$$

Thus, the apparent inconsistency arises from the fact that the asymptotic result due to Slepyan is valid over a region that becomes vanishingly small as $m \to 0$.

Graphs of the plastic strain distribution on the crack line in the active plastic zone are shown in Fig. 5.1 for $m = 0$, 0.3, 0.5. The plastic strain is singular in each case, as has already been noted. The most significant observation concerns the influence of material inertia on the strain distribution. An increase in crack speed results in a substantial reduction of the level in plastic strain for a fixed fractional distance from the crack tip to the elastic-plastic boundary. Therefore, if a local ductile crack growth criterion is imposed, then it would appear that the fracture resistance or toughness would necessarily increase with increasing crack tip speed. To quantify this idea, the fracture criterion proposed by McClintock and Irwin [49] was adopted. According to this criterion, a crack will grow with a critical value of plastic strain at a point on the crack line at a characteristic distance ahead of the tip. The crack will not grow for levels of plastic strain at this point below the critical level, and levels of plastic strain greater than the critical level are inaccessible. To make a connection between the plastic strain in the active plastic zone and the remote loading, a relationship between the size of the plastic zone and the remote applied stress intensity factor is required. This can be provided only through a complete solution of the problem, and it was obtained for the case of mode III by Freund and Douglas on the basis of a full field numerical solution of the governing equations. The resulting theoretical fracture toughness K_{IIId} versus crack speed curves are shown in Fig. 5.2 for three levels of the critical plastic strain $\gamma_c = 2\tau_o/\mu$, $5\tau_o/\mu$, $10\tau_c/\mu$. The critical distance has been eliminated in favor of K_{IIIc}, the level of applied stress intensity required to satisfy the same criterion for a stationary crack in the same material under equilibrium conditions. The different intercepts at $m = 0$ indicate an increasing propensity for stable crack growth with increasing toughness, and the intercept values correspond to the so-called steady state toughness values of the theory of stable crack growth.

The plots in Fig. 5.2 have some common general features. The ratio of K_{IIId}/K_{IIIc} is a monotonically increasing function of crack speed m which takes on large values for moderate values of m. Although there is no unambiguous way to associate a terminal velocity with these results, they suggest a maximum attainable velocity well below the elastic wave speed of the material. It is emphasized that the variation of toughness with crack speed in Fig. 5.2 is due to inertial effects alone. The material response is independent of rate of deformation, and the crack growth criterion that is enforced involves no characteristic time. If inertial effects were neglected, the calculated toughness would be *completely independent* of speed. The question of the influence of material rate sensitivity on this relationship is a separate issue.

The equivalent plane strain problem of dynamic crack growth in an elastic-ideally plastic material has not been so fully developed. However, a numerical calculation leading to a fracture toughness versus crack speed relationship, analogous to Fig. 5.2, has been described by Lam and Freund [50]. They adopted the critical crack tip opening angle growth criterion and derived results for mode I on the basis of the Mises yield condition and J_2 flow theory of plasticity that are quite similar in general form to those shown for mode III.

Viscoplastic Material Response

Consider steady crack growth in an elastic-plastic material for which the flow stress depends on the rate of deformation. The particular material model known as the over-stress power law model has been considered by a number of authors. According to this idealization, the plastic strain rate in simple shear $\dot{\gamma}^p$ depends on the corresponding shear stress τ through

$$\dot{\gamma}^p = \dot{\gamma}_t + \dot{\gamma}_o\{(\tau - \tau_t)/\mu\}^n \quad \text{for} \quad \tau \geq \tau_t \tag{5.3}$$

where $\dot{\gamma}_t$ is the threshold strain rate for this description, or the plastic strain rate when $\tau = \tau_t$. The description also includes the elastic shear modulus μ, the viscosity parameter $\dot{\gamma}_o$, and the exponent n. A common special case is based on the assumption that the slow loading response of the material is elastic-ideally plastic and that *all* inelastic strain is accumulated according to (5.3). For this case, $\dot{\gamma}_t = 0$ and τ_t is the slow loading flow stress τ_o. For other purposes, it is assumed that (5.3) provides a description of material response only for high plastic strain rates, in excess of the transition plastic strain rate $\dot{\gamma}_t$ and the corresponding transition stress level τ_t. For low or moderate plastic strain rates, the variation of plastic strain rate with stress is weaker than in (5.3), and a common form for the dependence is (cf. Frost and Ashby [51])

$$\dot{\gamma}^p = g_1(\tau)\exp\{-g_2(\tau)\} \tag{5.4}$$

where g_1 and g_2 are algebraic functions. The marked difference between response at low or moderate plastic strain rates and at high strain rates may be due to a change in fundamental mechanism of plastic deformation with increasing rate, or it may be a structure induced transition. For present purposes, it is sufficient to regard the difference as an empirical observation. The two forms of constitutive laws (5.3) and (5.4) can lead to quite different results in analysis of crack tip fields and, indeed, the form (5.3) leads to fundamentally different results for different values of the exponent n.

Lo [52] extended some earlier work on the asymptotic field for steady quasistatic crack growth in an elastic-viscoplastic material by Hui and Riedel [53] to included inertial effects. They adopted the multiaxial version of (5.3) with $\dot{\gamma}_t = 0$ and $\tau_t =$ to describe inelastic response, with no special provision for unloading. They showed that for values of the exponent n less than 3, the asymptotic stress field is the elastic stress field. For values of n greater than 3, on the other hand, Lo constructed an asymptotic field having the same remarkable feature of complete autonomy found by Hui and Riedel, that is, it show no dependence on the level of remote loading. For steady antiplane shear mode III crack growth, Lo found the radial dependence of the inelastic strain on the crack line ahead of the tip to be

$$\gamma_{yz}^{p}(x,0) \approx (n-1)\big(v/\dot{\gamma}_o x\big)^{1/(n-1)} T_L(v/c_s) \tag{5.5}$$

where the dependence of the amplitude factor T_L on crack speed is given graphically by Lo, who also analyzed the corresponding plane strain problem. Note that as $n \to \infty$ the plastic strain singularity becomes logarithmic. The full field solution for this problem under small scale yielding conditions was determined numerically by Freund and Douglas [54]. The numerical results showed a plastic strain singularity much stronger than for the rate independent case, and it appeared from the numerical results that the domain of dominance of the asymptotic field within the crack tip plastic zone expanded with increasing crack tip speed. These observations are consistent with (5.5).

The problem of steady growth of an antiplane shear crack in a strain rate sensitive elastic-plastic material was re-examined in a study by Yang and Freund [55]. The problem considered was the same as that studied by Lo [52] except that τ_t was taken to be the slow loading yield stress and the material was assumed to respond elastically with the initial shear modulus if it was unloaded. It was concluded in the earlier work on this problem that if the possibility of an elastic region near the crack tip was not considered then the asymptotic field was completely autonomous and the asymptotic solution could not be reduced to the generally accepted rate independent limit as the rate sensitivity of the material vanished. It was shown by Yang and Freund that if the possibility of elastic unloading was admitted in the formulation, then the asymptotic crack tip field does indeed approach the correct rate independent limit as the rate sensitivity vanishes. Furthermore, the existence of an elastic region at the crack tip provides a path for communication between the crack tip region and the remote loading, so that the crack tip field involves an undetermined parameter that can be determined only from remote fields.

High Strain Rate Crack Growth

A particularly interesting class of dynamic fracture problems are those concerned with crack growth in materials that may or may not experience rapid growth of a sharp cleavage crack, depending on the conditions of temperature, stress state and rate of loading. These materials may fracture by either a brittle or ductile mechanism on the microscale, and the focus is on establishing conditions for one or the other mode to dominate. The phenomenon is most commonly observed in ferritic steels. Such materials show a dependence of flow stress on strain rate, and the strain rates experienced by a material particle in the path of an advancing crack are potentially enormous. Consequently, the mechanics of rapid growth of a sharp macroscopic crack in an elastic-viscoplastic material that exhibits a fairly strong variation of flow stress with strain rate has been of interest in recent years. The general features of the process as experienced by a material particle on or near the fracture path are straight forward. As the edge of a growing crack approaches, the stress magnitude tends to increase there due to the stress concentrating effect of the crack edge. The material responds by flowing at a rate related to the stress level in order to mitigate the influence of the crack edge. It appears that the essence of cleavage crack growth is the ability to elevate the stress to a critical level before plastic flow can accumulate to defeat the influence of the crack tip. In terms of the mechanical fields near the edge of an advancing crack, the rate of stress increase is determined by the elastic strain rate, while the rate of crack tip blunting is determined by the plastic strain rate. Thus, an equivalent observation is that the elastic strain rate near the crack edge must dominate the plastic strain rate for sustained cleavage.

The problem has been studied from this point of view by Freund and Hutchinson [56]. They adopted the constitutive description (5.3,4) with $n = 1$. This is indeed a situation for which the near tip elastic strain rate dominates the plastic strain rate. Through an approximate analysis, conditions necessary for a crack to run at high velocity in terms of constitutive properties of the material, the rate of crack growth, and the overall crack driving force were extracted under small yielding conditions.

Consider the crack gliding along through the elastic-viscoplastic material under plane strain conditions. At points far from the crack edge, the material remains elastic and the stress distribution is given in terms of the applied stress intensity factor K_I by (2.4). Equivalently, the influence of the applied loading may be specified by the rate of mechanical energy flow into the crack tip region from remote points G, and these two measures are related by means of (4.3). For points near the crack edge the potentially large stresses are relieved through plastic flow, and a permanently deformed but unloaded wake region is left behind the advancing plastic zone along the crack flanks. For material particles in the outer portion of the active plastic zone the rate of plastic straining is expected to be in the low or moderate strain rate range, whereas for particles close to the crack edge, the response is modelled by the constitutive law (5.3) with $n = 1$. The stress distribution within this region then also has the form (2.4) but with a stress intensity factor *different* from the remote stress intensity factor. The crack tip stress intensity factor, say K_{Itip}, is assumed to control the cleavage growth process. The influence of the remote loading is screened from the crack tip by the intervening plastic zone, and the main purpose of the analysis is to determine the relationship between the remote loading and the crack tip field. For present purposes, it is assumed that the crack grows as a cleavage crack with a fixed level of local energy release rate, say G^c_{tip}. The question then concerns the conditions under which enough energy can be supplied remotely to sustain the level of energy release rate G^c_{tip} at the crack tip.

Recall that the energy flux integral (4.1) is path independent for any material response if the mechanical fields are steady, as in the present case. Thus, the matter of relating the applied G to G^c_{tip} was pursued by enforcing an overall energy rate balance by means of this integral. The balance may be cast into the form

$$G^c_{tip} = G - \frac{1}{v}\int_A \sigma_{ij}\dot{\epsilon}_{ij}\, dA - \int_{-h}^{h} U^*_e\, dy \tag{5.6}$$

where A is the area of the active plastic zone in the plane of deformation, h is the thickness of the plastic wake far behind the crack tip, and U^*_e is the residual elastic strain energy density trapped in the remote wake. This relation simply states that the energy being released from the body at the crack tip is the energy flowing into the crack tip region reduced by the energy dissipated through plastic flow in the plastic zone, and further reduced by the energy trapped in the wake due to incompatible plastic strains. The expression is exact.

Through several approximations, the complete energy balance (5.6) was reduced to the remarkably simple form

$$G/G^c_{tip} = 1 + D(m)P_c \tag{5.7}$$

where the dimensionless parameter P_c is $\dot{\gamma}_o\sqrt{\mu\rho}\,G^c_{tip}(1 + 2\dot{\gamma}_t\mu/\dot{\gamma}_o\tau_t)/3\tau_t^3$ and $D(m)$ is a dimensionless function of crack tip speed $m = v/c_r$. P_c is a monotonically increasing function of temperature for steels with values in the range from about 0 to 10 as temperature varies from $0K$ to about $400\,K$. The function $D(m)$ is asymptotically unbounded as $m \to 0$ and $m \to 1$, and it has a minimum at an intermediate crack tip speed. A graph of $D(m)$ is shown in Fig. 5.3.

For any given value of temperature or, equivalently, for any value of P_c, the graph in Fig. 5.3 gives the locus of pairs G, v for which steady state propagation of a sharp crack can be sustained. The implication is that if a cleavage crack can be initiated for a pair G, v that is *above* the curve, then the crack will accelerate to a state on the stable branch of the curve. If the driving force diminishes as the crack advances, or if the local material temperature increases as the crack advances, then the state pair will move toward the minimum point on the curve. If the driving force is further decreased, or if the temperature is further increased, then further growth of a sharp cleavage crack cannot be sustained according to the model. The implication is that the crack will arrest abruptly from a fairly large speed, and a plastic zone will then grow from the arrested crack. Further crack growth is possible if either a ductile growth criterion can be met or if cleavage can be reinitiated through strain hardening in the evolving plastic zone. The details of the model have been refined through full numerical solution of the problem [57], but the essential features have not changed with more precise analysis.

6. EXPERIMENTAL OBSERVATIONS

The subject of experimental methods and results is far too vast and substantive to be summarized with balance here. Instead, some recent experimental findings that are connected in one way or another with the theoretical ideas introduced in the foregoing sections are briefly mentioned.

The initiation of crack growth resulting from loading a planar crack by means of a plane tensile pulse of finite duration was studied by Shockey, Kalthoff and Erlich [58]. This was achieved by loading disks of a brittle epoxy with well characterized internal cracks in a plate impact device. The amplitude and duration of the loading pulse and the size of the internal cracks were controlled. It was found that the attainment of a critical stress intensity factor level was not sufficient to initiate growth of the internal cracks. Instead, they reported that it was necessary for the dynamic stress intensity factor to *exceed* the fracture toughness for some short time before crack growth would commence.

The same issue of initiation of crack growth due to pulse loading in a material due to stress pulse loading was studied by Smith and Knauss [59] who simulated the situation of a semi-infinite crack in an unbounded body. Their specimen was a sheet of the brittle polymer Homalite-100, large enough so that waves reflected from the outer boundaries returned only after the experiment was over. A sharp crack was cut into the sheet from one edge and a copper ribbon was folded over into the cut. A large stored electrical charge was then discharged through the ribbon, and the resulting induced mechanical forces provided essentially uniform pressure loading on the crack faces. They also found that the apparent level of stress intensity at onset of crack growth seemed to exceed the fracture toughness, with the difference increasing with the intensity of crack face pressure. The experiment was refined and new data were reported subsequently by Ravi-Chandar and Knauss [60]. In both cases, the crack tip stress intensity factor was measured directly by means of the shadow spot method in transmission mode for the transparent material.

Ravi-Chandar and Knauss [61] modified their set-up to simulate the situation of an opposed pair of point loads acting on opposite crack faces as analyzed by Freund [13]. Again, the transient stress intensity factor was measured in a large sheet of Homalite-100 by means of the optical shadow spot method, and the experimental results and theoretical prediction were in good agreement. A similar situation was studied by Kim [62] who used a novel optical method of measurement of the transient stress intensity factor history to verify some rather unusual features in the analytical solution of the model problem. Both Ravi-Chandar and Knauss [61] and Kim [62] also studied the crack propagation phase that followed initiation in their experiments.

An experiment has been developed by Ravi-Chandran and Clifton [63] to permit the study of fracture initiation and crack propagation in metal specimens under intense stress pulse loading. The specimen is in the form of a disk with a midplane fatigue crack grown in so that its edge is on a diameter of the disk. The specimen is impacted by a flyer plate in a gas gun which produces a compressive plane pulse that travels through the plane of the closed crack without modification. The pulse reflects as a tensile pulse from the traction free back face of the specimen and causes fracture initiation within a few hundred nanoseconds of its arrival at the crack plane. The configuration has the feature of being a half plane crack in an unbounded body near the center of the specimen, at least until unloading waves from the boundaries penetrate this region. The amplitude of the stress pulse is determined by the impact velocity and the duration of the pulse is determined by the thickness of the flyer plate. Thus, the situation is a realization of the model discussed in section 3 above. Preliminary experiments done with 4340 steel specimens in a very hard condition ($R_C = 55$) suggest that crack speeds that are a significant fraction of the Rayleigh wave speed of the material can be achieved.

Jumps in stress intensity factor showing the general trend deduced from analysis of crack propagation and arrest in the string model have been observed experimentally by Kalthoff, Winkler and Beinert [64] in double cantilever beam specimens of a brittle plastic material, Araldite B. The specimens were loaded quasi-statically in the wedge loading arrangement, and the stress intensity factor history was monitored by means of the optical shadow spot method. The initial crack tip was slightly blunted, and the sharp crack that grew from the blunted pre-crack experienced a decreasing driving force as it advanced.

The crack speed did not remain constant up to arrest as predicted by the string model, of course, but it did remain relatively constant for some time, gradually decelerating before arrest. The experiments played an important role in dynamic fracture research because they showed conclusively that specimen inertia can have a significant role in the dynamic crack growth process.

Some data on the dynamic fracture toughness of metals during rapid crack growth are available. Rosakis, Duffy and Freund [65] used the optical shadow spot method in reflection mode to measure the prevailing stress intensity factor during rapid crack growth in 4340 steel hardened to $R_C = 45$. This is a relatively strain rate insensitive material with very little strain hardening, so that the material may presumably be modeled as elastic-ideally plastic. The observed toughness varied little with crack speed for speeds up to about 600 to 700 m/s, and thereafter the toughness increased sharply with increasing crack tip speed. The general form of the toughness versus speed data was similar to the theoretical prediction based on the numerical simulation reported by Lam and Freund [50], lending support to the view that material inertia on the scale of the crack tip plastic zone has an important influence on the perceived dynamic fracture toughness. Similar data were reported by Kobayashi and Dally [66] who made photoelastic measurements of the crack tip stress field by means of a birefringent coating on the specimen. Data on crack propagation and arrest in steels were reported by Dahlberg, Nilsson and Brickstad [67].

Important experiments on crack propagation and arrest in steel specimens are currently being carried out by deWit and Fields [68]. Their specimens are enormous single edge notched plates loaded in tension. The growing crack thus experiences an increasing driving force as it advances through the plate. A temperature gradient is also established in the specimen so that the crack grows from the cold side of the specimen toward the warm side. Based on the presumption that the material becomes tougher as the temperature is increased, the crack also experiences increasing resistance as it advances through the plate. The specimen material is A533B pressure vessel steel, which is both very ductile and strain rate sensitive. In the experiments, the fracture initiates as a cleavage fracture and propagates at high speed through the specimen into material of increasing toughness. The crack then arrests abruptly in material whose temperature is *above* the nil ductility temperature for the material based on Charpy tests. A large plastic zone grows from the arrested crack edge, and cleavage crack growth is occasionally reinitiated. The essential features of the experiment appear to be consistent with the model of high strain rate crack growth discussed in section 5, and this model appears to provide a conceptual framework for interpretation of the phenomenon. An analysis of rapid crack growth in a rate dependent plastic solid has also been carried out by Brickstad [69] in order to interpret some experiments on a high strength steel.

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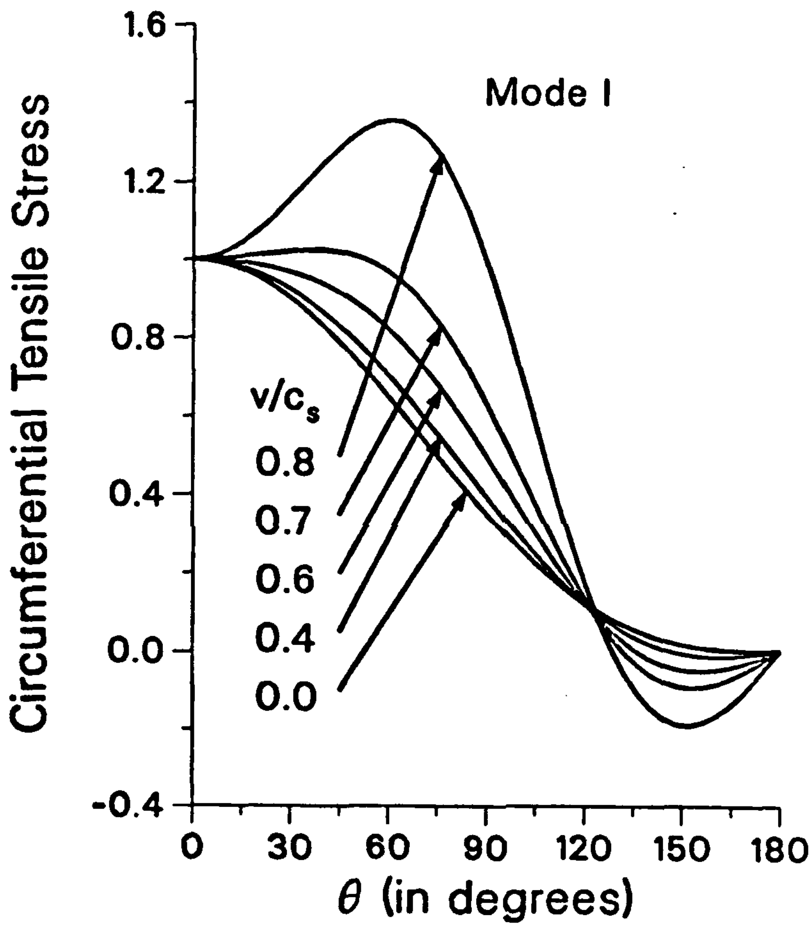

Fig. 2.1 Angular variation of the circumferential tensile stress for the asymptotic field (2.4), normalized with respect to $K_I/\sqrt{2\pi r}$.

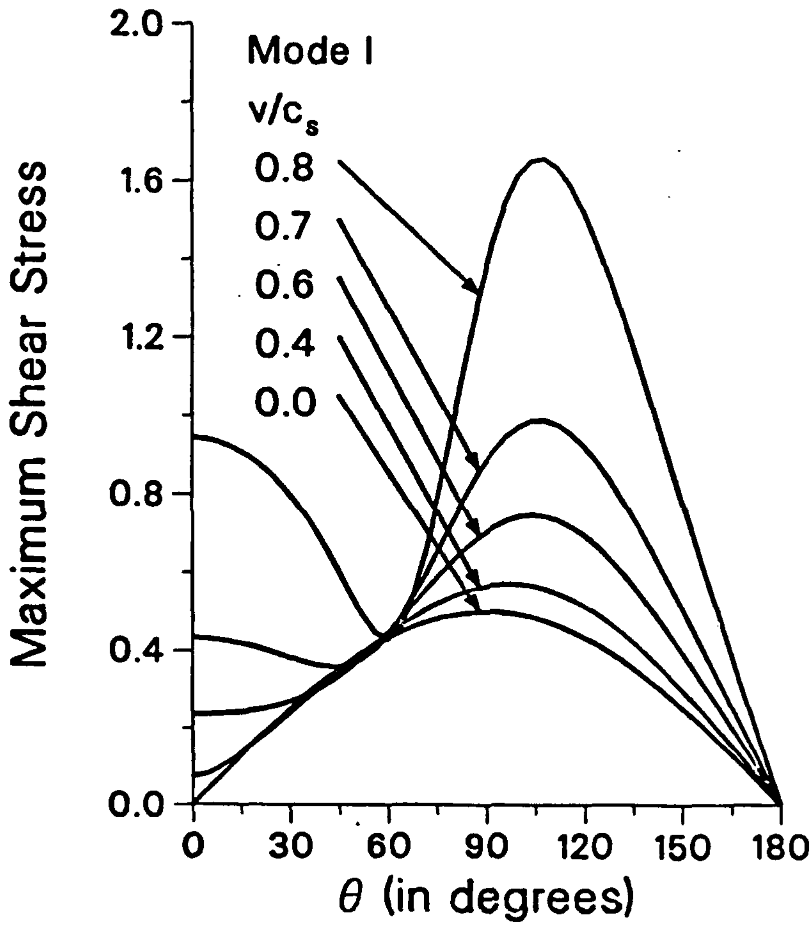

Fig. 2.2 Angular variation of the maximum shear stress for the asymptotic field (2.4), normalized with respect to $K_I/\sqrt{2\pi r}$.

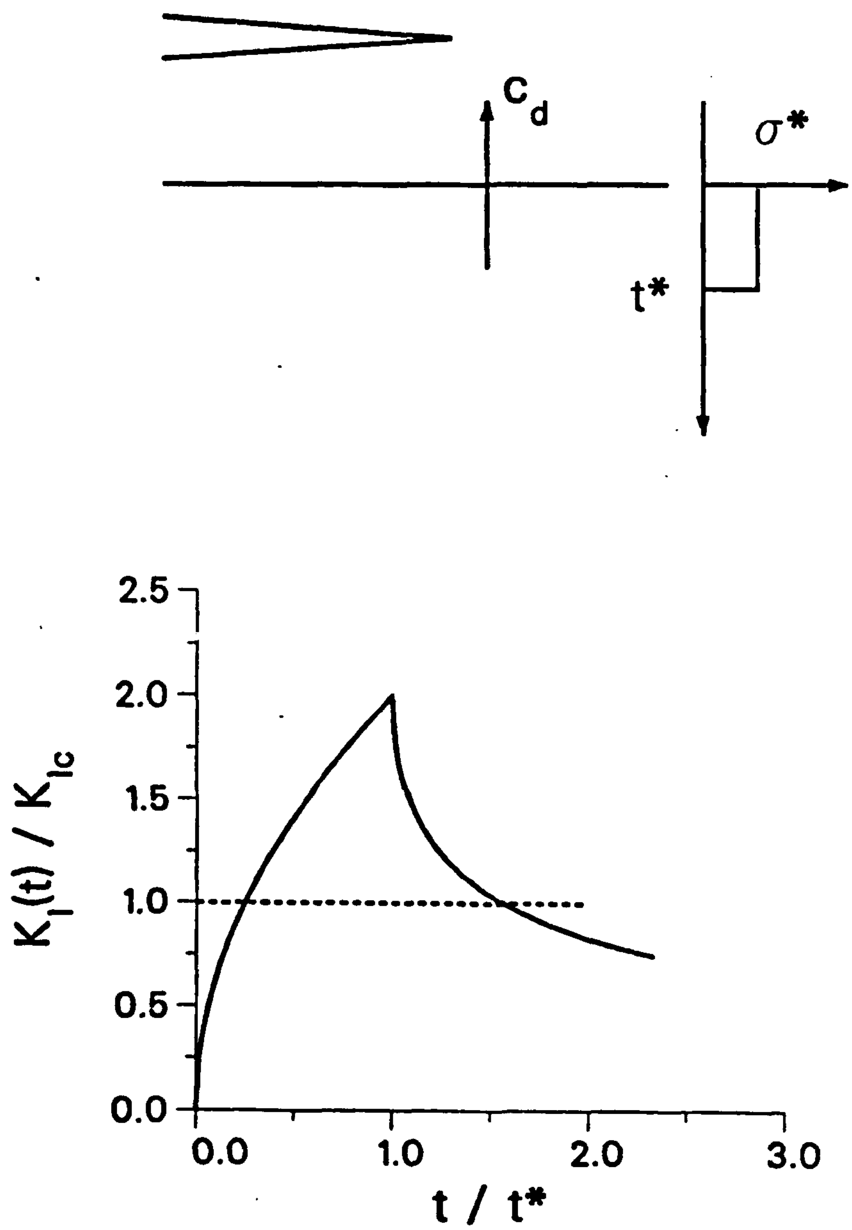

Fig. 3.1 Time history of the stress intensity factor for diffraction of a plane tensile pulse of magnitude σ^* and duration t^*.

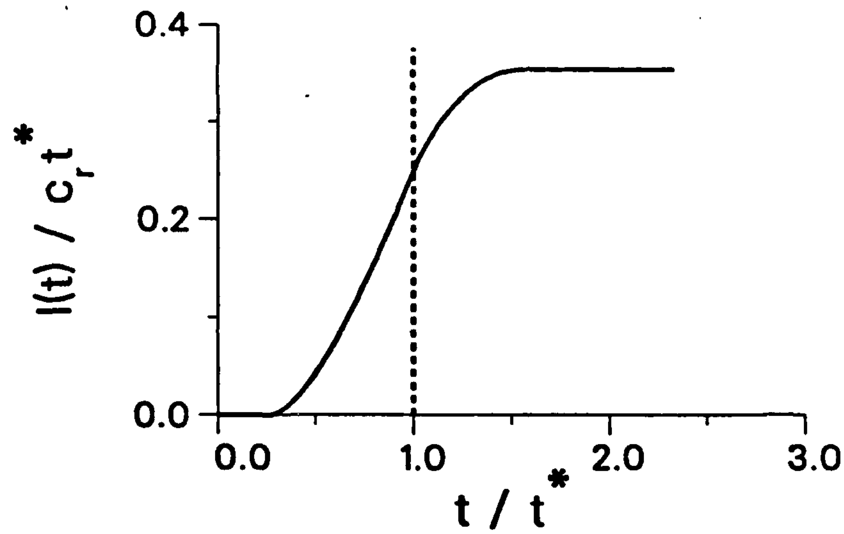

Fig. 3.2 Crack tip motion implied by the differential equation (3.6) for the generalized Irwin growth criterion.

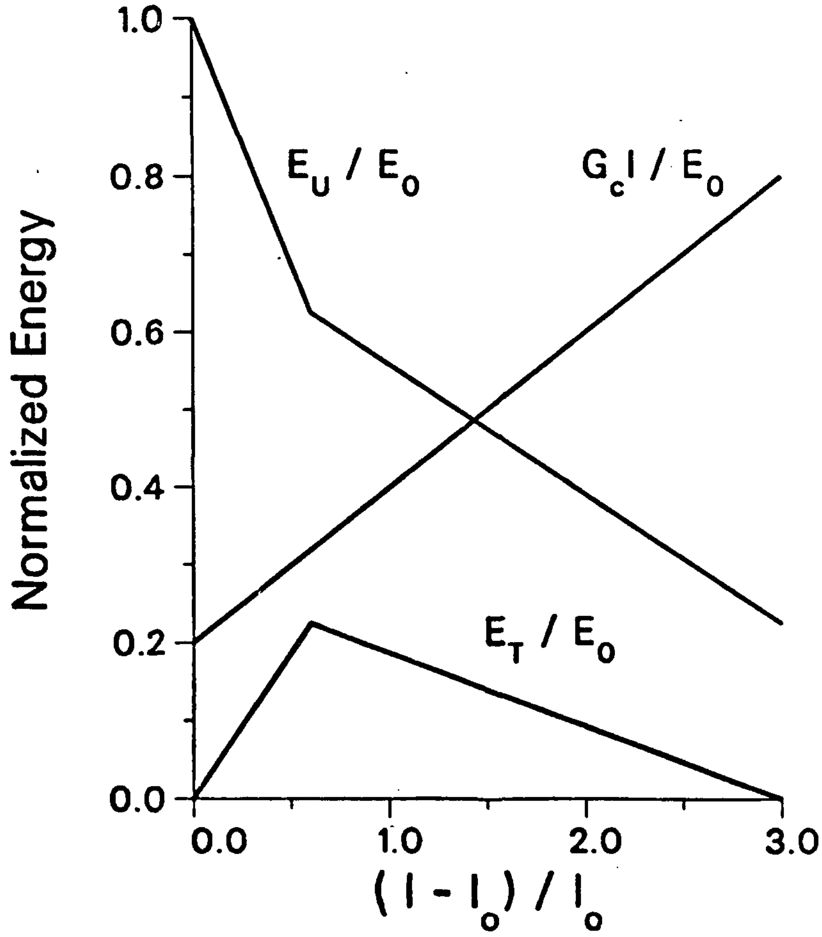

Fig. 4.1 Variation of energy measures with crack length for the one dimensional crack growth model.

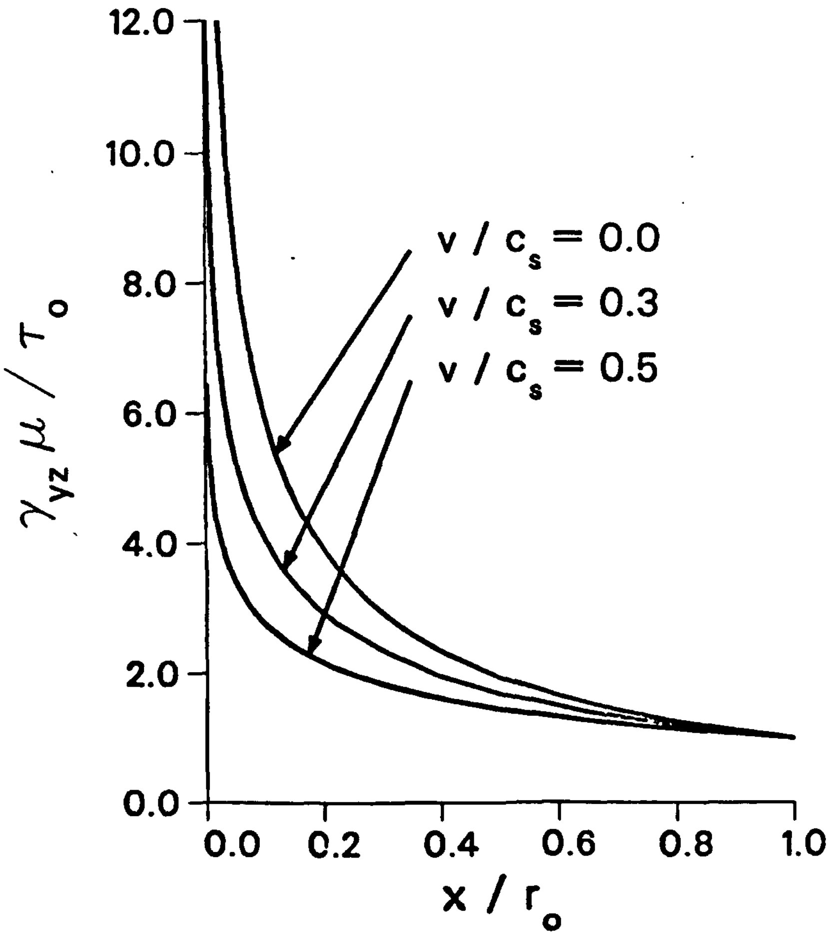

Fig. 5.1 Total strain on the crack line in the active plastic zone for steady dynamic growth of a mode III crack in an elastic-ideally plastic material from (5.1).

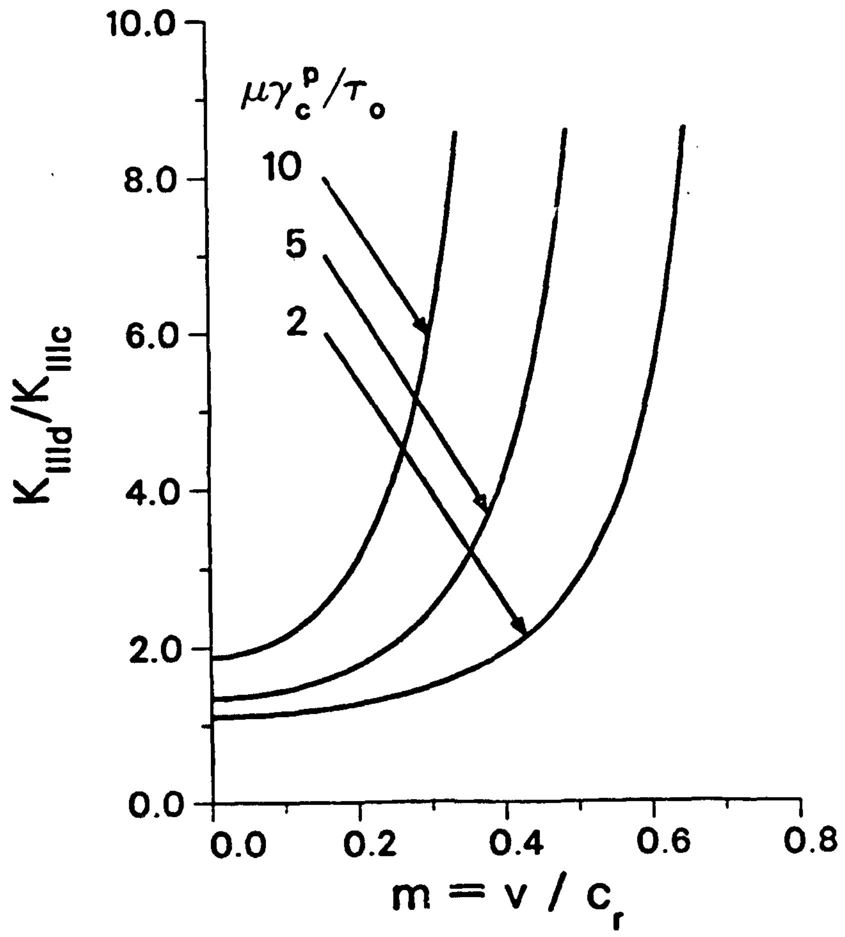

Fig. 5.2 Theoretical fracture toughness versus crack speed for steady growth of a mode III crack according to the critical plastic strain at a characteristic distance criterion, for three levels of critical plastic strain.

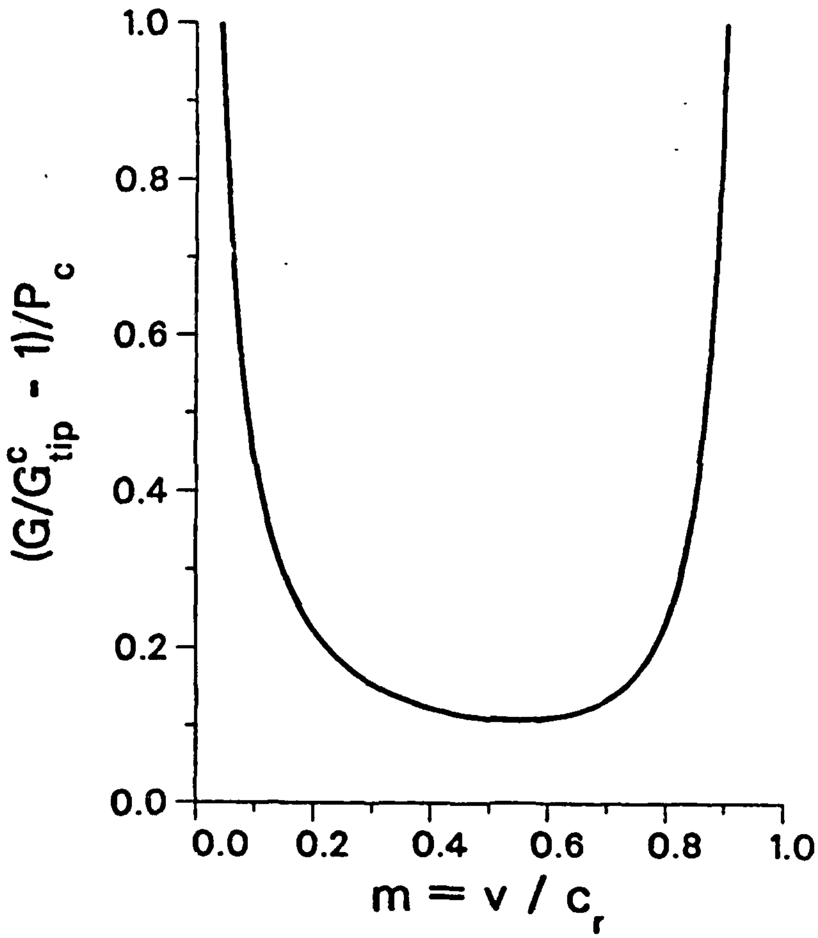

Fig. 5.3 Graph of the function $D(m) = (G/G^{c}_{tip} - 1)/P_c$ from (5.7).